Neutrino production, oscillation and detection in the presence of general four-fermion interactions

Matthew Garbutt & Bruce H. J. McKellar
School of Physics, University of Melbourne, Victoria, Australia 3010

28 July 2003

Abstract

In this note we investigate the impact of new physics in the form of general four-fermion interactions on neutrino oscillation signals. We develop a field theoretic description of the overall oscillation process which includes the non-standard interactions during the neutrino production, propagation and detection stages. Insights gained during the development of this formalism regarding the possibility of new interactions mimicking neutrino mass differences are expounded. The impact of possible new physics is assessed by studying the $\nu_\mu \to \nu_\tau$ oscillation channel in vacuum and the $\nu_e \to \nu_\mu$ channel in matter. Although it is known that the effects of new interactions can only act as a perturbation to the leading oscillation parameters, we find that great care needs to be taken when drawing conclusions regarding the strength of new couplings from oscillation measurements.

1 Introduction

The observation of neutrino oscillations have had a profound impact on our understanding of particle physics. The measurement of atmospheric neutrino oscillations at the Superkamiokande (SK) experiment indicates that ν_μ's oscillates to ν_τ's with a mass squared difference of $\approx 3 \times 10^{-3}\text{eV}^2$ [1], a result confirmed by K2K [2]. The simplest interpretation of this result is that at least one of the neutrino mass eigenstates has a non-zero mass of $m \geq 5.5 \times 10^{-2}\text{eV}$, and as such provides the first glimpse of physics beyond the Standard Model (SM). Recent measurements of the solar neutrino spectrum at the Sudbury Neutrino Observatory (SNO) have confirmed the solar neutrino results of SK and found that ν_e's from the sun are oscillating to the $\nu_\mu - \nu_\tau$ subsystem [3, 4, 5]. The corresponding mass squared difference is smaller.

The standard treatment of neutrino mixing has three active flavour eigenstates mixed with three mass eigenstates. This mixing is described by the rotation of one basis into the other by the angles θ_{13}, θ_{12} and θ_{23} and a complex phase. The atmospheric data constrains $\sin\theta_{23}$ to be almost maximal with a mass squared difference of $|\delta m_{13}| \approx 3\times10^{-3}\text{eV}^2$, while the solar neutrino data also constrains θ_{12} to be large and has $|\delta m_{12}| \leq 3 \times 10^{-4}\text{eV}^2$ [6]. These "large mixing angle results" for the solar neutrino mixing were recently confirmed by the terrestrial Kamland experiment [7]. Furthermore θ_{13} is bound by reactor data such that $\sin^2\theta_{13} \leq 0.1$ [8].

The signs of the mass differences are still unknown, while information on possible complex CP violating phases is extremely scant. In order to overcome this there have been a number new experimental facilities proposed such as the the JHF neutrino beam, and others based on a muon storage ring. All provide a high intensity neutrino beam over a wide range of base lengths and energies [9, 10, 11].

Alongside the neutrino oscillation industry there has been an ongoing and long established program of precision experiments aimed at measuring the properties of the weak interaction. Measurement of the Tritium beta decay spectrum end-point is one such program while the determination of the Michel parameters in the μ and τ-decay spectra is another [12, 13, 14, 15, 16, 17, 18]. Originally the focus of these experiments was to establish the Lorentz structure of the weak interaction, that is to decide whether it is Vector minus Axial-Vector (V-A) or some other combination of Scalar and Tensor operators [19]. Now that the V-A structure has been established the focus

has shifted to measuring the electron neutrino mass, at least in the case of the Tritium experiments [20]. Not until very lately have the possible effects of neutrino mixing on these experiments been considered [21, 22, 23, 24].

In this paper the converse scenario is examined, that is the impact of non-standard interactions, such as additional Lorentz structures in the weak interaction, on future neutrino oscillation experiments. The aim of this work is to derive a formalism that will describe the production, propagation and detection of a massive neutrino with general Lorentz structures present at each of these stages. In effect we aim to incorporate the couplings that comprise the Michel parameters, and the β-decay parameters of Ref. [19] into the description of neutrino oscillations. Similar calculations have focussed on specific extensions to the Standard Model, or have been performed exclusive of one or more of the production, propagation or detection process. The correspondence between these previous calculations and this work is expounded.

2 Background to NSI

In the SM weak processes are mediated via the exchange of a charged vector boson, and are successfully described at low energies by a $V-A$ current-current interaction with an effective coupling strength G_F. Presently the SM has withstood all experimental tests with the exception of the recently observed apparent non-zero mass of the neutrino. Despite this there are many theories that seek to extend the SM, often motivated by the desire to restore a broken symmetry of the SM or to unify the strong force with the electroweak force, or even to unite all four of the fundamental forces under one theory. Usually this unification occurs at a higher mass scale than the SM, that is the bosons that mediate the new interactions have a mass larger than the W and Z bosons of the electroweak theory. The new interactions will manifest themselves as current-current theories at low energies in the same way as the SM charged current reduces to Fermi's effective field theory for energies significantly less than the mass of the Z-boson.

If new physics is present at higher mass scales the low energy result will be to induce new effective interactions with a coupling strength weaker than the coupling of the dominant $V-A$ interaction unless the fundamental coupling constant is anomolously large. Absent a specific model there is no reason to expect that the new physics has the same Lorentz structure as the weak interaction, hence new currents may be a scalar (S), pseudo-scalar (P) or tensor (T) in nature or comprise of different combinations of vector and axial vector operators. Given the dominance of the left-handed $V-A$ interaction we find it convenient to cast these new interactions into forms with definite handedness[1], $L=V-A$, $R=V+A$, $S_L=S-P$ and $S_R=S+P$.

Assuming lepton universality the most general interaction Hamiltonians for low energy, leptonic and semi-leptonic processes are given by

$$H_L = \sum_{\alpha\beta} g^{\alpha\beta}\bar{\psi}_l\Gamma_\beta\psi_{\nu_l^{\alpha\beta}}\left(\bar{\psi}_m\Gamma_\alpha\psi_{\nu_m^{\alpha\beta}}\right)^\dagger + h.c., \tag{1}$$

and,

$$H_{SL} = \sum_{\alpha\beta} G^{\alpha\beta}\bar{\psi}_l\Gamma_\beta\psi_{\nu_l^{\alpha\beta}}\left(\bar{\psi}_q\Gamma_\alpha\psi_{q'}\right)^\dagger + h.c. \tag{2}$$

respectively. Where $\alpha/\beta = L,\ R,\ S_L,\ S_R$, while ψ_q and $\psi_{q'}$ are quark fields, ψ_l and ψ_m are charged lepton fields of flavour l and m respectively. The neutrino field is $\psi_{\nu_l^{\alpha\beta}}$, representing a neutrino produced by an ($\alpha\beta$) type interaction and associated with a charged lepton of flavour l. The operators Γ_λ are a combination of one of the five bilinear covariants

$$\begin{aligned}
\Gamma_L &= \gamma_\nu(1-\gamma_5)\ , \\
\Gamma_R &= \gamma_\nu(1+\gamma_5)\ , \\
\Gamma_{S_L} &= (1-\gamma_5)\ , \\
\Gamma_{S_R} &= (1+\gamma_5)\ .
\end{aligned} \tag{3}$$

[1] To be completely general one would need to also consider new V-A interactions in addition to the standard interaction. This has been the subject of numerous investigations, see Ref [25][26][27].

In the SM only $\alpha = (L, R)$ and $\beta = L$ are present in the semi-leptonic case with $\psi_{\nu_l^{LL}} = \psi_{\nu_l^{RL}}$. And for the standard leptonic interaction only $\alpha = L$ and $\beta = L$ are allowed.

The fact that neutrinos oscillate indicates that the interaction eigenstate is not the same as the mass eigenstate. Typically the two are related via a unitary transformation

$$\psi_{\nu_f} = \sum_i U_{fi}\psi_{\nu_i} \ . \tag{4}$$

Phenomenologically, there is no reason why a neutrino produced by one interaction needs to be coupled to the same combination of mass eigenstates as a neutrino produced by another interaction. So in this work neutrino mixing is described by

$$\psi_{\nu_f^{\alpha\beta}} = \sum_i U_{fi}^{\alpha\beta}\psi_{\nu_i} \ . \tag{5}$$

Or equivalently each new interaction has its own unitary mixing matrix which may be equivalent, or not, to the standard mixing matrix.

The Eqs. 1 & 2 describe the production and detection of a neutrino associated with a charged lepton, in addition Eq. 1 can describe the interaction of a neutrino, via a charged current, with a background medium of charged leptons. This interaction leads to the well known resonant enhancement of the neutrino mass, the MSW effect, crucial to the solution of the solar neutrino problem [28, 29]. In order to be used in the derivation of the matter induced potential, to which we will return in a later section, Eq. 1 needs to be cast in the form of a neutral current. This is achieved by Fierz rearrangement (See for example Ref. [30]) such that

$$\begin{aligned} -\mathcal{L} &= \sum_{\alpha,\beta} g_{lf}^{\alpha\beta} \left(\bar{\psi}_l \Gamma^\alpha \psi_{\nu_l}^{\alpha\beta}\right)^\dagger \bar{\psi}_f \Gamma^\beta \psi_{\nu_f}^{\alpha\beta} \\ &= \sum_{\alpha,\beta}\sum_{ij} K_{(l)ij}^{\alpha\beta} \bar{\psi}_{\nu_j}\Gamma_\beta \psi_{\nu_i} \left(\bar{\psi}_l \Gamma^\alpha \psi_l\right)^\dagger \end{aligned} \tag{6}$$

where

$$\begin{aligned} K_{(l)ij}^{LL} &= -g^{LL}U_{li}^{LL}U_{lj}^{LL}, \\ K_{(l)ij}^{RR} &= -g^{RR}U_{li}^{RR}U_{lj}^{RR}, \\ K_{(l)ij}^{RL} &= \frac{1}{2}g^{S_LS_L}U_{li}^{S_LS_L}U_{lj}^{S_LS_L}, \\ K_{(l)ij}^{LR} &= \frac{1}{2}g^{S_RS_R}U_{li}^{S_RS_R}U_{lj}^{S_RS_R}, \\ K_{(l)ij}^{S_LS_L} &= \frac{1}{2}g^{S_RS_L}U_{li}^{S_RS_L}U_{lj}^{S_RS_L} + 3g^{T_RT_L}U_{li}^{T_RT_L}U_{lj}^{T_RT_L}, \\ K_{(l)ij}^{S_RS_R} &= \frac{1}{2}g^{S_LS_R}U_{li}^{S_LS_R}U_{lj}^{S_LS_R} + 3g^{T_LT_R}U_{li}^{T_LT_R}U_{lj}^{T_LT_R}, \\ K_{(l)ij}^{S_RS_L} &= 2g^{RL}U_{li}^{RL}U_{lj}^{RL}, \\ K_{(l)ij}^{S_LS_R} &= 2g^{LR}U_{li}^{LR}U_{lj}^{LR}, \\ K_{(l)ij}^{T_LT_L} &= \frac{1}{4}g^{S_RS_L}U_{li}^{S_RS_L}U_{lj}^{S_RS_L} - \frac{1}{2}g^{T_RT_L}U_{li}^{T_RT_L}U_{lj}^{T_RT_L}, \\ K_{(l)ij}^{T_RT_R} &= \frac{1}{4}g^{S_LS_R}U_{li}^{S_LS_R}U_{lj}^{S_LS_R} - \frac{1}{2}g^{T_LT_R}U_{li}^{T_LT_R}U_{lj}^{T_LT_R} \ . \end{aligned} \tag{7}$$

This potential has been written in the mass basis so that Eq. 6 actually describes the interaction of a mass eigenstate, rather than a flavour eigenstate, as is usually presented. The effective coupling constant $K_{(l)ij}^{\alpha\beta}$ is dependent on the type of leptons in the background as it contains the various mixing elements specific to the lepton flavor. We do not included a neutral current term for two reasons. Firstly, the standard neutral current contributes only to the absolute value of the effective neutrino masses and not to the mass differences or mixing angles. And secondly, any non-standard neutral current is highly constrained by experimental data from atomic physics and LEP.

In this work we apply the formalism being developed presently to terrestrial experiments, hence a background of charged fermions only is considered. In the general case one would also need to consider the effects of forward scattering off a neutrino background. This has important consequences in early universe physics, described for example in [31, 32, 33].

As alluded to earlier the Lorentz structure of the various currents in the effective Lagrangians presented above may be generated by the presence of physics beyond the SM. One prominent example is the low energy effects of Supersymmetry [34]. In particular R-parity violating modes lead to charge-changing scalar currents [35]. In the model discussed in Ref. [35] stau mediated lepton number violating decay, $\mu^- \to e^-\nu_e\bar{\nu}_\mu$, is examined. For instance the correspondence between the Lagrangian that produces this decay and Eq. 1 may be found by setting $\psi_{\nu_e^{S_L S_L}} \equiv \psi_{\nu_\mu^{LL}}$ and $\psi_{\nu_\mu^{S_L S_L}} \equiv \psi_{\nu_e^{LL}}$ and if for the supersymmetric coupling the following substitution is made $\lambda_{132}\lambda_{231}/(s - \tilde{m}_\tau^2) \equiv G^{S_L S_L}$, where $\tilde{m}_\tau$ is the stau mass and λ_{132} and λ_{231} are coupling constants. Supersymmetry aside, right-handed vector currents can be motivated by left-right symmetric models where a heavier W boson couples to right-handed neutrinos, while scalar, vector and tensor currents may arise from Leptoquark theories [36].

3 Formalism—Scattering theory

Spectral measurements such as the Tritium and Michel parameter experiments yield little information about the mixing elements $U_{fi}^{\alpha\beta}$, hence it is natural to look to oscillation experiments to access these parameters. To do this a formalism that describes the oscillation of a neutrino with information about both the production and detection processes is needed. To do this we treat the whole production, propagation and detection process as a single scattering event. In this scenario the neutrino and charged vector bosons at the production and detection sites are treated as unobserved intermediate states. The oscillation phenomena arises through the interference between these scattering diagrams. Now if NSI are present additional diagrams with the non-standard boson at the production and/or detection site need to be included.

Field theoretic (scattering theory) calculations of neutrino oscillations have been performed by a number of authors [37, 38, 39, 40, 41]. For a complete list of references and a thorough review see the work by Beuthe, Ref. [37]. In this note the formalism developed by Cardall & Chung, Ref. [31] is adapted to cater for NSI.

To keep the notation transparent the formalism will be developed for a specific example, that of μ^+-decay at the source, electron neutrino[2] oscillation and subsequent detection of a negatively charge muon at the detector. The scattering amplitude is given by the time ordered product of interaction Hamiltonians

$$\begin{aligned} A &= <\bar{\nu}_\mu e^+ X \mu^- | T[\int d^4x \int d^4y H_L(x) H_{SL}(y)] | \mu^+ N> \\ &= \int_{-\infty}^{\infty} dx^0 \int_{-\infty}^{\infty} dy^0 \int_{V_S} d^3x \int_{V_D} d^3y \frac{1}{\sqrt{(2\pi)^6 V_S^3 V_D^3}} \\ &\times \frac{\exp[-i(p_{\mu^+} - p_e - p_{\bar{\nu}})\cdot x]}{\sqrt{(2E_{\mu^+})(2E_e)(2E_{\bar{\nu}})}} M_{fi}(x,y|p_i,p_f,q) \frac{\exp[-i(p_N - p_{\mu^-} - p_X)\cdot y]}{\sqrt{(2E_N)(2E_{\mu^-})(2E_X)}} , \end{aligned} \tag{8}$$

where E_{μ^+}, E_e and $E_{\bar{\nu}}$, are the energies of the anti-muon, electron and the muon anti-neutrino (not participating in the oscillation measurement) with corresponding four momenta p_{μ^+}, p_e and $p_{\bar{\nu}}$. The reaction at the source is evaluated at a space-time point x in a volume V_S. The energies of the particles at the detector are E_N, E_{μ^-}, E_X with associated four-momenta p_N, p_{μ^-}, p_X. The reaction at the detector is evaluated at a space-time point y in a volume V_D. Furthermore the matrix element $M_{fi}(x,y|p_i,p_f,q)$ is given by

$$\begin{aligned} M_{fi}(x,y|p_i,p_f,q) &= \sum_{ij}\sum_{\alpha\beta}\sum_{\lambda\sigma} g^{\alpha\beta} G^{\lambda\sigma} U_{\mu i}^{\alpha\beta} U_{\mu j}^{\lambda\sigma} J^\lambda(p_N,p_X)^\dagger \\ &\times \left(\bar{u}(p_{\mu^-})\Gamma_\sigma G^{ji}(x,y)\gamma^0\Gamma_\alpha^\dagger\gamma^0 v(p_e)\right)\cdot\left(\bar{v}(p_{\mu^+})\Gamma_\beta v(p_{\bar{\nu}})\right) . \end{aligned} \tag{9}$$

[2] The term electron neutrino is used loosely here since neutrinos produced via different interactions and associated with the decay of muons are are not necessarily the same. This is due to the various mixing matrices associated with the different interactions. When it matters more specific language will be used.

Here p_i and p_f are the momenta of the particles in the initial and final state respectively, q is the momentum of the oscillating neutrino while $J^\lambda(p_N, p_X)$ is the nuclear matrix element for the transition operator Γ_λ. The sums are over neutrino vacuum masses, source and detector interactions respectively. The function $G^{ij}(x,y)$ is the neutrino propagator or Green's function. In vacuum, $G^{ij}(x,y) \to G^{ii}(x,y)$ and is given by the standard Dirac propagator.

The starting point in the evaluation of $G^{ij}(x,y)$ is the equation of motion for a propagating neutrino from which the equation satisfied by the neutrino Green's function can be found,

$$(i\partial\!\!\!/ - M - V)G(y,x) = \delta^4(y-x) \ , \tag{10}$$

where V is the propagation potential derived from Eq. 6 and M is the mass matrix, note the mass indices have been suppressed as we will be concerned with $G(y,x)$ in spinor space for the moment. Due to the chiral nature of the operators Γ_σ and $\Gamma^\dagger_\alpha$ it is convenient to write the Green's function in chiral blocks

$$G(x,y) = \begin{pmatrix} G_{LL}(x,y) & G_{LR}(x,y) \\ G_{RL}(x,y) & G_{RR}(x,y) \end{pmatrix} \ , \tag{11}$$

where G_{XY} is the element projected out by $P_X G(x,y) P_Y$ where $P_X = P_{R/L} = \frac{1}{2}(1 \pm \gamma_5)$. Similarly the operator in Eq. 10 in chiral form can be written as

$$(i\partial\!\!\!/ - M - V) = \begin{pmatrix} -M_1 & \sigma\cdot(i\partial - V^{RR}) \\ \bar\sigma\cdot(i\partial - V^{LL}) & -M_2 \end{pmatrix} \ , \tag{12}$$

where

$$M_1 \ = \ M + S^{S_R S_L} \tag{13}$$

and

$$M_2 \ = \ M + S^{S_L S_R} \ , \tag{14}$$

with

$$S^{S_R S_L} = \sum_l \int \frac{d^3p}{(2\pi)^3}\rho_l(\vec p_l)\frac{m_l}{E_l}(K^{S_R S_L} + K^{S_L S_L}) \ , \tag{15}$$

$$S^{S_L S_R} = \sum_l \int \frac{d^3p}{(2\pi)^3}\rho_l(\vec p_l)\frac{m_l}{E_l}(K^{S_L S_R} + K^{S_R S_R}) \ , \tag{16}$$

$$(V^{LL})^\mu = \sum_l \int \frac{d^3p}{(2\pi)^3}\rho_l(\vec p_l)\frac{p_l^\mu}{E_l}(K^{LL} + K^{RL}) \ , \tag{17}$$

$$(V^{RR})^\mu = \sum_l \int \frac{d^3p}{(2\pi)^3}\rho_l(\vec p_l)\frac{p_l^\mu}{E_l}(K^{RR} + K^{LR}) \ . \tag{18}$$

The sums in Eqs. 15-18 are over lepton species with mass m_l distributed according to $\rho_l(\vec p_l)$ in the background medium. In general the Green's function may be expressed in any basis, however we find it convenient to do so in the basis defined by the vacuum mass eigenstates. In this basis the matrix M is diagonal, while the matrices $S^{S_R S_L}$, $S^{S_L S_R}$, $(V^{LL})^\mu$ and $(V^{RR})^\mu$ are in general not diagonal. All other operators are proportional to the unit in this basis assuming the potential V does not vary in space or time.

Using Eq. 11 & Eq. 12 two sets of two equations can be found and used to solve for each component of the Green's function. For G_{LR} and G_{RR};

$$\delta^4(x_2 - x_1) \ = \ -M_2 G_{RR} + \left[i(\partial_0 - \vec\sigma\cdot\vec\nabla) - V^{LL}\cdot\bar\sigma\right] G_{LR} \ , \tag{19}$$

$$0 \ = \ -M_1 G_{LR} + \left[i(\partial_0 + \vec\sigma\cdot\vec\nabla) - V^{RR}\cdot\sigma\right] G_{RR} \ , \tag{20}$$

and for G_{LL} and G_{RL};

$$\begin{aligned}
\delta^4(x_2-x_1) &= -M_1 G_{LL} + \left[i(\partial_0+\vec{\sigma}\cdot\vec{\nabla}) - V^{RR}\cdot\sigma\right] G_{RL} \ , & (21)\\
0 &= -M_2 G_{RL} + \left[i(\partial_0-\vec{\sigma}\cdot\vec{\nabla}) - V^{LL}\cdot\bar{\sigma}\right] G_{LL} \ . & (22)
\end{aligned}$$

For the analysis that is to follow in the next section only the first set of equations will need to be solved. This is done following Cardall & Chung, Ref. [31], by defining

$$J(x_2,x_1) = M_1^{-1} G_{RR}(x_2,x_1), \tag{23}$$

which reduces the problem to solving

$$\left(-M_2M_1 + [i\bar{\sigma}.\partial - v_L\cdot\bar{\sigma}]\,[i\sigma\cdot\partial - v_R\cdot\sigma]\right) J(x,y) = \delta^4(y-x) \ , \tag{24}$$

where $v_R = M_1^{-1}V^{RR}M_1$ and for brevity $v_L = V^{LL}$. Now the delta function and $J(y,x)$ are expanded into momentum space via a Fourier transform where we find that

$$\big(-M_2M_1 + [(q_0+\vec{\sigma}\cdot\vec{q}) - v_L\cdot\bar{\sigma}]\,[(q_0-\vec{\sigma}\cdot\vec{q}) - v_R\cdot\sigma]\big) J(q_0,\vec{q}) = 1 \ . \tag{25}$$

Where q_0 and $\vec{q}$ are the unobserved neutrino energy and momenta. The task is now to find the elements of $J(q_0,\vec{q})$ in terms of its reciprocal matrix $R(q_0,\vec{q}) = J(q_0,\vec{q})^{-1}$. Recalling that $J(q_0,\vec{q})$ is a matrix in the 2×2 space of the Pauli spinors, one can make a substantial simplification by choosing the direction of neutrino propagation to coincide with third spatial coordinate. For a non-polarised, non-relativistic background of leptons we find that:

$$\begin{aligned}
R(q_0,\vec{q})_{11} &= q_0^2 - |\vec{q}|^2 - M_2M_1 - \left[q_0 v_R^0 + v_R^0|\vec{q}|\right] \\
&- \left[q_0 v_L^0 - v_L^0|\vec{q}|\right] + v_L^0 v_R^0 \ , & (26)\\
R(q_0,|\vec{q}|)_{12} &= 0 \ , & (27)\\
R(q_0,|\vec{q}|)_{21} &= 0 \ , & (28)\\
R(q_0,|\vec{q}|)_{22} &= q_0^2 - |\vec{q}|^2 - M_2M_1 - \left[q_0 v_R^0 - v_R^0|\vec{q}|\right] \\
&- \left[q_0 v_L^0 + v_L^0|\vec{q}|\right] + v_L^0 v_R^0 \ , & (29)
\end{aligned}$$

where the subscripts correspond to the elements of the matrix $R(q_0,\vec{q})$.

To perform the Fourier transform back to coordinate space the pole structure of $J(q_0,\vec{q})$ must be established. This is accomplished by defining the unitary matrices $\tilde{U}_R$ and $\tilde{U}_L$ which diagonalize, in flavour space, R_{11} and R_{22} respectively. In the ultra-relativistic limit where $|\vec{q}| \to q_0$, the components of $J(x_2,x_1)$ are

$$\begin{aligned}
J(\vec{y},\vec{x})_{11} &= \int \frac{dq_0}{2(2\pi)^2}\frac{e^{-iq_0(y^0-x^0)}}{|\vec{y}-\vec{x}|}\sum_K e^{-iq_0|\vec{y}-\vec{x}|}\tilde{U}^R_{iK}\tilde{U}^R_{jK}e^{i\frac{m^2_{RK}}{2q_0}|\vec{y}-\vec{x}|} & (30)\\
J(\vec{y},\vec{x})_{22} &= \int \frac{dq_0}{2(2\pi)^2}\frac{e^{-iq_0(y^0-x^0)}}{|\vec{y}-\vec{x}|}\sum_K e^{-iq_0|\vec{y}-\vec{x}|}\tilde{U}^L_{iK}\tilde{U}^L_{jK}e^{i\frac{m^2_{LK}}{2q_0}|\vec{y}-\vec{x}|} \ . & (31)
\end{aligned}$$

The effective masses m^2_{RK} and m^2_{LK} are the K eigenvalues of the matrices $-M_2M_1 + v^0m^0 - 2q_0m^0$ and $-M_2M_1 + v^0m^0 - 2q_0v^0$ respectively.

The components of the Green's function, G_{RR} and G_{LR}, are found by substitution into Eq. 23 and then Eq. 19. In the matrix element, Eq. 9, the sums over vacuum masses are carried out such that

$$\sum_i\sum_K U^{\alpha\beta}_{\mu i}U^R_{iK} \to \sum_K U^{\alpha\beta R}_{\mu K} \tag{32}$$

where the sum is now over effective masses.

The scattering amplitude is found by localising the particles at the source and detector to boxes of volume V_S and V_D and by expanding $\vec{x}$ and $\vec{y}$ about the source and detection points $\vec{x}_S$ and $\vec{y}_D$ respectively [31]. Finally we find for the square of the scattering amplitude,

$$\begin{aligned} \mathcal{A}_L \mathcal{A}_K^* &= T \frac{1}{{V_S}^2} \frac{1}{{V_D}^2} \left[(2E_{\mu^+})(2E_e)(2E_{\bar{\nu}})\right]^{-1} \left[(2E_N)(2E_X)(2E_{\mu^-})\right]^{-1} \\ &\times \ (2\pi)^4 \delta^4(p_{\mu^+} - p_e - p_{\bar{\nu}} - q)(2\pi)^3 \delta^3(\vec{p}_N + \vec{q} - \vec{p}_{\mu^-} - \vec{p}_X) \\ &\times \ \left(\frac{1}{4\pi|\vec{L}|}\right)^2 \exp[i\delta m_{ji}^2 |\vec{L}|](\mathcal{M}_{\mu^+}^L \mathcal{M}_{\mu^+}^{K*})(\mathcal{M}_{\mu^-}^L \mathcal{M}_{\mu^-}^{K*}) \ . \end{aligned} \tag{33}$$

To find a per source particle per detector particle event rate integrate over the final state phase space (factor of $Vd^3p/(2\pi)^3$ for each particle in the final state) and divide by T after interpreting one of the energy delta functions as characterising the time of the scattering process;

$$d\Gamma(E_q) = \int \sum_{KL} \frac{\exp(i\delta m_{KL}^2/2E_q|\vec{L}|)}{|\vec{L}|^2} \left.\frac{d^2N_\nu}{dE_q d\Omega_q}\right|_{KL} \sigma(\nu N)_{KL} dE_q \ , \tag{34}$$

The quantity $d^2N_\nu/dE_q d\Omega_q$ is interpreted as the neutrino flux;

$$\left.\frac{d^2N_\nu}{dE_q d\Omega_q}\right|_{KL} = \int \frac{d^3p_e}{(2\pi)^3} \frac{d^3p_{\bar{\nu}}}{(2\pi)^3} \frac{(2\pi)^4 \delta^4(p_{\mu^+} - p_e - p_{\bar{\nu}} - q) E_q^2 \mathcal{M}_{\mu^+}^{L*} \mathcal{M}_{\mu^+}^K}{(2\pi)^3 (2E_{\mu^+})(2E_q)(2E_{\bar{\nu}})(2E_e)} \ , \tag{35}$$

where the (KL) dependence arises from the various mixing elements in matter. The (νN) cross section also has a (KL) dependence and is given by

$$\sigma(\nu N)_{KL} = \int \frac{d^3p_X}{(2\pi)^3} \frac{d^3p_{\mu^-}}{(2\pi)^3} \frac{(2\pi)^4 \delta^4(p_N + q - p_X - p_{\mu^-}) \mathcal{M}_{\mu^-}^L \mathcal{M}_{\mu^-}^{K*}}{(2E_q)(2E_N)(2E_X)(2E_{\mu^-})} \ . \tag{36}$$

The mixing elements are all contained in the squared matrix elements and are not in general able to be extracted and interpreted as an oscillation probability, except in the case where only one type of interaction is responsible for source and detection processes, or if only one type of mixing matrix exists, i.e. $U^{\alpha\beta} \equiv U^{\gamma\delta}$.

The event rate is obtained by integrating over the source and detector particle distributions:

$$\frac{dN}{dE_q} = \int d^3x_S \int \frac{d^3p_{\mu^+}}{(2\pi)^3} f_\mu(\vec{p}_{\mu^+}, \vec{x}_S) \int d^3y_D \int \frac{d^3p_N}{(2\pi)^3} f_N(\vec{p}_N, \vec{y}_D) \frac{d\Gamma}{dE_q} \ . \tag{37}$$

This event rate, with Eqs. 20 and 22, is the first main result of this paper. It is an expression incorporating neutrino oscillations in matter, and after following the same procedure just outlined to solve for G_{LR} & G_{LL}, allows for arbitrary Lorentz couplings. There are three main points of difference between this result and the standard result. Firstly, in vacuum there is the possibility of observing flavour violation as a result of the intermediate neutrino even if the neutrino mass differences are zero. This requires that the non-standard mixing matrix not be related trivially to the standard matrix. In this scenario no oscillatory phase will develop. Secondly, in matter, the addition of NSI allows for an oscillatory phase even if the neutrino masses vanish. Again the non-standard mixing matrix must not be trivially related to the standard matrix. These results were first noted by Bergman *et. al.* in Ref. [42] and subsequently investigated by Huber *et. al.* in Ref. [43]. They found that NSI could only act as a perturbation to the solution of the solar and atmospheric problems rather than a complete explanation. And thirdly, we note that the presence of a right-handed potential V^{RR} connects a left-handed neutrino state to a right handed state, even in the ultra-relativistic limit. This is akin to the spin flip caused by a non-zero neutrino magnetic moment in a strong field. We do not pursue this here, however we note that the presence of this potential in a dense medium will require the equations of motion for right-handed and left-handed neutrinos be solved simultaneously, perhaps using a density matrix approach. Finally we highlight the fact that if the right-handed potential v_R is zero then the only non-zero component of the Greens function is G_{22} in the ultra-relativistic limit. Thus the connection to right-handed states vanishes and the only required diagonalization matrix is $\tilde{U}^L$.

4 Considerations for a neutrino beam

The formalism developed in the previous section can now be applied to a generic neutrino factory scenario. In this section no attempt is made at a full simulation of this type of experiment, A full simulation would involve a discussion of detector properties, efficiencies, energy resolutions, cuts, backgrounds and the like; rather we attempt to define regions of interesting parameter space and achieve a quantitative understanding of the effects of NSI. As we have seen, in the relativistic limit, the event rate for a particular experiment can be obtained by evaluating the neutrino flux, $d^2N_\nu/dE_q d\Omega_q|_{KL}$, containing the effects of NSI at the source, the production cross section, $\sigma(\nu n)_{KL}$, accounting for NSI at the detector, and the phase factor $\exp(i\delta m^2_{KL}|\vec{L}|/2E_q)/|\vec{L}|^2$ dealing with the oscillatory phase. In this paper we present the results of two investigations. Firstly the vacuum propagation of a neutrino produced at the source via μ^--decay and subsequent production of a τ^- at the detector, that is $\nu_\mu \to \nu_\tau$ oscillation. Secondly we examine the case of a neutrino produced via μ^+-decay, interacting with the electrons in the background medium, and producing a μ^- in the detector, or $\nu_e \to \nu_\mu$ oscillation. In both cases this is performed for the simple case of one non-standard scalar coupling, $(\alpha, \beta) = (S_L, S_L)$, in addition to the standard interaction in Eqs. 1 & 2. This particular non-standard coupling is chosen since it is the only interaction other than the standard one that does not require the existence of right-handed neutrinos. The strength of the coupling for both the leptonic and semi-leptonic interaction is taken to be $g^{S_L S_L} = G^{S_L S_L} = 0.01\ G^{LL}$, within the current experimental upper bounds [44, 45].

The expression for the neutrino flux resulting from the decay of a μ^- can be calculated using standard trace techniques, in general one will find two new terms as a result of the scalar interaction, an interference term between the standard and non-standard interaction of order $|g^{S_L S_L}|$ and a pure scalar term of order $|g^{S_L S_L}|^2$. The interference term reflects the right-handed admixture, proportional to $\frac{m_\mu}{E_\mu}$, in the left-handed wavefunction of the μ^- [46]. The chiral structure of the scalar interaction being considered requires a right-handed μ^-, while the standard weak interaction always proceeds via a left-handed μ^-. Since the process by which muons will be produced at a neutrino factory will be dominated by the standard weak interaction, e.g. π-decay, any contribution from NSI will be doubley suppressed, once by the production process and once by the decay process. For this reason we neglect the contribution from NSI at the source. However the expression for the neutrino flux with the additional NSI terms is recorded here for completeness. In the rest frame of the muon, neglecting all lepton masses we find that

$$\left.\frac{d^2N_\nu}{dx d\Omega_q}\right|_{ij} = \omega_\mu \frac{2x^2}{4\pi}\left[U^{LL}_{\mu j}U^{LL}_{\mu i}(3-2x) + 3\rho^2 U^{S_L S_L}_{\mu j}U^{S_L S_L}_{\mu i}(1-x)\right], \tag{38}$$

where $x = 2m_\mu/E_\nu$ and $\rho = G^{LL}/G^{S_L S_L}$.

For energies above the tau threshold of $\sim$ 5GeV the dominant reaction mechanism at the detector is deep inelastic scattering producing many particles in the final state in addition to the tau, denoted collectively as X. The $(\nu_\tau N)$ cross section can be derived using a parton model of the nucleon. In the standard treatment the lepton and parton masses are typically assumed to be zero, this is a good approximation in most circumstances however its validity for this application is questionable. This is due in part to the considerable mass of the τ and also to the fact that certain mixing patterns may act to pick out the interference term in the cross section which would otherwise vanish in the massless limit as we shall show. A treatment using massive partons has been given by Aivazis *et. al.* in Ref [47], and the non-standard cross section is derived using this formalism.

The general expression for the cross section with a vanishing neutrino mass is

$$\begin{aligned} d\sigma &= \frac{2\pi Q^2}{\Delta(s,0,M^2)}\Big(|G^{LL}|^2 L^{\alpha\beta}W_{\alpha\beta} \\ &+ 2G^{LL}G^{S_L S_L}L^\alpha W_\alpha + |G^{S_L S_L}|^2 LW\Big)d\Gamma, \end{aligned} \tag{39}$$

where $\Delta(s, 0, M^2)$ is a kinematic function of the Mandelstam variable s and the nucleon mass M, while we have retained a non-zero τ-mass we have neglected the small neutrino mass. The

final state phase space is represented by $d\Gamma$. The complete evaluation of Eq. (39) is lengthy but involves only standard trace algebra, here we just state the results. The cross section in terms of the Bjorken scaling variables x and y is written as

$$\begin{aligned} \left.\frac{d\sigma}{dxdy}\right|_{ij} &= U_{\tau i}^{LL}U_{\tau j}^{LL}\left[\frac{d\sigma}{dxdy}\right]_{LL} + U_{\tau j}^{S_L S_L}U_{\tau i}^{S_L S_L}\left[\frac{d\sigma}{dxdy}\right]_{SS} \\ &+ \frac{1}{2}\left(U_{\tau j}^{LL}U_{\tau i}^{S_L S_L} + U_{\tau j}^{S_L S_L}U_{\tau i}^{LL}\right)\left[\frac{d\sigma}{dxdy}\right]_{SL} \end{aligned} \tag{40}$$

where the first two terms arise from vector and scalar interactions respectively and the final term arises from the interference between the two. In the limit where all terms of order m_i^2/Q^2 or greater can be ignored (where in this context m_i corresponds to the parton masses) we find

$$\left[\frac{d\sigma}{dxdy}\right]_{SS} \approx |G^{S_L S_L}|^2 \frac{ME_\nu}{2\pi}\left(xy^2 + \frac{m_\tau^2 y}{2ME_\nu}\right)F_1(x), \tag{41}$$

$$\left[\frac{d\sigma}{dxdy}\right]_{SL} \approx G^{S_L S_L}G^{LL}\frac{ME_\nu}{2\pi}m_\tau\left(\frac{1}{E_\nu} - (\frac{xy}{E_\nu} + \frac{m_\tau^2}{2ME_\nu^2})\right)F_1(x), \tag{42}$$

The term, $[d\sigma/dxdy]_{LL}$, not shown here is the standard results and was derived in Ref [48]. The structure functions, F_1 through F_5 are measured quantities fitted to a functional form, we use the same form as is used in Ref [49]. In the limit that $m_1^2/Q^2 \to 0$ the struck parton mass becomes $m_1 \to xM$ as noted in Ref [50].

With the assumption of an idealised source of neutrinos arising from muon decay the charged current event rate can be written as

$$\begin{aligned} N_\tau &= 6.023 \times 10^{32}\frac{N_\mu M_{kt}}{E_\mu|\vec{L}|^2} \\ &\times \int\left[\sum_{ij}\left.\frac{d^2\rho_\nu^*}{dxd\Omega_q}\right|_{ij}\sigma_{\nu N}|_{ij}\exp(-i\frac{\delta m_{ij}^2|\vec{L}|}{2E_q})\right]dE_q\ , \end{aligned} \tag{43}$$

where $\omega_\mu d^2\rho_\nu^*/dxd\Omega_q = d^2N_\nu^*/dxd\Omega_q$ is the neutrino distribution in the reference frame of the lab. For high energy muons $x \to E_q/E_\mu$ and:

$$\begin{aligned} \frac{d^2\rho_\nu^*}{dxd\Omega_q} &= \frac{1}{\gamma^2(1-\beta\cos\alpha)^2}\frac{d^2\rho_\nu}{dxd\Omega_q} \\ &\approx 4\gamma^2\frac{d^2\rho_\nu}{dxd\Omega} \end{aligned} \tag{44}$$

where $\alpha = 0$ coincides with the beam direction, $\gamma = E_\mu/m_\mu$ and $\beta = p_\mu/E_\mu$. For high beam energies $\cos\alpha \sim 1$ and $\beta \sim 1 - 1/(2\gamma)$. The number of useful muon decays per year, N_μ, is defined as the integration over the source distribution times the μ-decay rate. The parameter M_{kt} is the mass of the detector in kilotons. The numerical factor in Eq. (43) is the number of nucleons in the detector per kiloton, we have assumed an ideal detector comprising $6.023\times10^{32}M_{kt}$ non-relativistic nucleons.

5 Vacuum oscillations—Two neutrinos.

We now study the effects of varying the non-standard mixing angle for the $\nu_\mu \to \nu_\tau$ vacuum oscillation channel.

The study is based on the philosophy that the atmospheric, solar and Kamland neutrino data sets have accurately defined the leading oscillation parameters. That is the LMA solution is accurate and the NSI are treated as perturbations to this solution. This philosophy is backed by the study of atmospheric neutrinos and NSI in Ref [51]. We examine a simplified two neutrino

system through the $\nu_\mu \to \nu_\tau$ oscillation channel. As such the standard and non-standard mixing matrices are defined as

$$U^{LL} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \text{ and } U^{S_L S_L} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} . \tag{45}$$

The standard mixing angle and mass difference given by the LMA solution are $\sin^2(2\theta) \approx 1$ and $\delta m^2 \approx 2.5 \times 10^{-3}$ eV^2.

The baselines over which it is proposed that oscillation experiments will be conducted vary from a few hundred kilometres to several thousand. For example at the proposed Fermilab neutrino factory there are plans to place detectors 732 km away at the Soudan detector and even at the South Pole a distance of 11700 km. The energies of the muon beam used to produce the neutrino flux are to be optimised for the detection of CP violation in the neutrino sector. Typically energies of $20-50$ GeV are being studied. The great advantage using a muon storage ring as a neutrino source is the extremely high intensity of the resulting beam with $\sim 10^{21}$ neutrinos expected to be produced per year.

Other studies have investigated the effects on oscillations of varying the base length $|\vec{L}|$ and the beam energy E_q with NSI present [25]. In any case variations in the beam length will have little consequence for the parameter range we will consider; that is for energies, mass difference and base lengths where $\delta m^2_{ij}|\vec{L}|/2E_q << 1$. For the $\nu_\mu \to \nu_\tau$ oscillation channel in the absence of NSI

$$\begin{aligned} P_{\mu\tau} &\approx 2(\cos\theta\sin\theta)^2 - 2(\cos\theta\sin\theta)^2\left(1-\frac{1}{2}\left(\frac{\delta m^2_{ij}|\vec{L}|}{2E_q}\right)^2\right) \\ &= (\cos\theta\sin\theta)^2\left(\frac{\delta m^2_{ij}|\vec{L}|}{2E_q}\right)^2 . \end{aligned} \tag{46}$$

When this is substituted into an event rate such as Eq. (43) the dependence on $|\vec{L}|$ is removed.

The sensitivity of a future neutrino factory to NSI is investigated by defining a χ^2 function which determines the required detector mass and number of useful muon decays in order to claim new physics. The χ^2 function assuming Gaussian statistics for a detector mass and muon number of $N_\mu \cdot M_{det} = 10^{21}$kt/yr is defined as

$$\chi^2_{NM21} = \sum_k \frac{|N_k^{SM} - N_k^{NSI}|^2}{N_k^{SM}} , \tag{47}$$

where N_k^{SM} is the expected number of τ producing charged current events in energy bin k in the absence of NSI and for this detector mass and muon number

$$\begin{aligned} N_k^{SM} &= 6.023 \times 10^{32}\frac{N_\mu M_{kt}}{E_\mu|\vec{L}|^2} \\ &\times \int_{E_{k-1}}^{E_k}\left[\sum_{ij}\left.\frac{d^2\rho^*_\nu}{dx d\Omega_q}\right|_{ij} d\sigma_{\nu N}|_{ij}\exp(-i\frac{\delta m^2_{ij}|\vec{L}|}{2E_q})\right] dE_q . \end{aligned} \tag{48}$$

Here the energy bins are defined such that $E_{k-1} < E_q < E_k$ $(k = 1, 2, 3, .., n)$. Furthermore N_k^{NSI} is the number of charged current events in the k^{th} energy bin expected with the NSI present. We define a required detector mass-muon number unit, $NM_{rec} = 1 \times 10^{21}$ kt/yr, to quantify the sensitivity to NSI for a given set of parameters. The constraint on NM_{rec} is

$$NM_{rec} > \frac{\chi^2_{90\%}}{\chi^2_{NM21}}, \tag{49}$$

where $\chi^2_{90\%}$ is the χ^2 value at 90% confidence level and one detector mass-muon number unit defined as a function of the number of degrees of freedom[3]. The sensitivity of NM_{rec} to the number of

[3]The subtleties of performing this kind of analysis have been examined in Ref. [25].

energy bins used, where the number of bins corresponds to the number of degrees of freedom, was investigated by varying the number of bins for various values of beam energy. We found the result to be statistically stable for ten bins or greater.

In Fig. 1 the variation of NM_{rec} with respect to the non-standard mixing angle ϕ is shown. The first plot is for a beam energy of $E_\mu = 50$ GeV , while the second is for a beam energy of $E_\mu = 20$ GeV. Points of particular interest are $\phi = \pi/4,\ 5\pi/4$ and $\phi = 3\pi/4,\ 7\pi/4$. The first case corresponds to the situation $\nu_\mu^{S_L S_L} \equiv \nu_\mu^{LL}$ or $U_{\mu i}^{S_L S_L} \equiv U_{\mu i}^{LL}$ up to a phase. For this case NM_{rec} is close to a maximum. In the second case $\nu_\mu^{S_L S_L} \equiv \nu_\tau^{LL}$ or $U_{\mu i}^{S_L S_L} \equiv U_{\tau i}^{LL}$ up to a phase, these points correspond to direct flavour violation. In this case NM_{rec} is almost at a minimum. This becomes obvious when we consider the mixing of the non-standard basis within the standard by performing the rotation $(\nu_\alpha^{LL}) = U^{LL\dagger} U^{S_L S_L} (\nu_\alpha^{S_L S_L}) = W(\nu_\alpha^{S_L S_L})$; now

$$\begin{pmatrix} \nu_\mu^{LL} \\ \nu_\tau^{LL} \end{pmatrix} = \begin{pmatrix} c_\theta c_\phi + s_\theta s_\phi & c_\theta s_\phi - s_\theta c_\phi \\ s_\theta c_\phi - c_\theta s_\phi & s_\theta s_\phi + c_\theta c_\phi \end{pmatrix} \begin{pmatrix} \nu_\mu^{S_L S_L} \\ \nu_\tau^{S_L S_L} \end{pmatrix}. \tag{50}$$

For the case of $\sin(2\theta) = 1$ the standard basis coincides with the non-standard when $\cos(\phi) - \sin(\phi) = 0$ up to a phase and is directly flavour violating when $\cos(\phi) + \sin(\phi) = 0$.

The two plots also exhibit a strong energy dependence with a much richer structure evident for the higher beam energy. This effect may be understood through a careful examination of the $(\nu_\tau N)$ cross section. The interference term is suppressed by a factor of E_q^{-1} relative to the other terms, thus its importance is enhanced at low energies. In Fig. 2 the relative contributions to the event rate from the pure scalar and the interference terms are shown. The first plot is with a beam energy of $E_\mu = 50$ GeV and the second with $E_\mu = 20$ GeV. At high energies the magnitude of the contribution from the pure scalar term is approximately equivalent to that of the interference term, this is despite the fact that it is suppressed by a relative factor of $G^{S_L S_L}$. At the lower beam energy the interference term is not yet washed out and dominates over the pure scalar term.

This simplified analysis indicates that for some values of ϕ a detector mass of ~ 1000kt would be in a good position to either detect NSI or increase the upper bound on the non-standard coupling strength. This result is to be expected, since at this energy and over a medium base line the neutrino beam will comprise mostly of non-oscillated ν_μ's. Any flavour violating non-standard coupling will in essence be picked out over the standard term. This example effectively demonstrates the convenience of this formalism, it allows one to obtain results for flavour violating couplings, as in Ref. [35], or flavour diagonal couplings simply by dialling up the appropriate value of ϕ. It also serves as a useful reminder that absent a specific SM extension the phase of a new interaction can play as important a role as the coupling strength in oscillation experiments, while it is unimportant in spectral tests of NSI.

6 Matter enhanced oscillations—two neutrinos

We now move on to examine the impact of NSI on the experimental signature of the matter enhanced $\nu_e \to \nu_\mu$ oscillation channel. Again this study is conducted with a generic neutrino factory in mind. For this calculation only new interactions during the propagation stage of the oscillation process are considered, no new interaction are present at the detector or the source. In particular a left-chiral scalar interaction coupled with a strength of $g^{S_L S_L} = 0.01 g^{LL}$ is examined. Since there is no new physics at the detector or the source it is possible to factor out an 'oscillation probability' in Eq. 43. We will comment further on this issue later.

The propagation potential for a neutrino with the SM and left-chiral scalar interaction present is given by Eq. (7) and Eq. (17) in the mass basis as

$$(V^{LL})^\mu \cdot \gamma_\mu (1-\gamma_5) \quad = \quad \sqrt{2} G_F U_{ei}^{LL} U_{ej}^{LL} n_e - \frac{g^{S_L S_L}}{g^{LL}\sqrt{2}} G_F U_{ei}^{S_L S_L} U_{ej}^{S_L S_L} n_e \tag{51}$$

or in the SM interaction basis

$$(V^{LL})^\mu \cdot \gamma_\mu (1-\gamma_5) \quad = \quad \sqrt{2} G_F n_e - \sum_{\alpha\beta} \frac{g^{S_L S_L}}{g^{LL}\sqrt{2}} G_F W_{\alpha\tilde{e}} W_{\beta\tilde{e}} n_e \tag{52}$$

where $W = U^{LL\dagger}U^{S_LS_L}$ was introduced in Eq. 50. We have denoted the non-standard basis with the tilde in Eq. (52). The propagation potential is rotated into the SM basis for ease of comparison with the standard treatment of matter enhanced oscillations, in this spirit the matrix W is parameterised in an analogous way to the non-standard vacuum mixing matrix:

$$W = \begin{pmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{pmatrix} . \tag{53}$$

With these considerations the Hamiltonian for neutrinos in matter is

$$\tilde{H} = \frac{1}{2E_q}\left[U^{LL}\begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix}U^{LL\dagger} - W\begin{pmatrix} \frac{A'}{2} & 0 \\ 0 & 0 \end{pmatrix}W^\dagger + \begin{pmatrix} A & 0 \\ 0 & 0 \end{pmatrix}\right] \tag{54}$$

where $A' = 2\sqrt{2}\frac{g^{S_LS_L}}{g^{LL}}G_F n_e E_q$ and $A = 2\sqrt{2}G_F n_e E_q$. The expression $\tilde{H}$ can be rewritten as

$$\begin{aligned} \tilde{H} &= \frac{1}{4E_q}\Bigg[\Sigma + A - \frac{A'}{2} + \frac{A'}{2}\begin{pmatrix} -\cos(2\psi) & \sin(2\psi) \\ \sin(2\psi) & \cos(2\psi) \end{pmatrix} \\ &+ A\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \delta m^2\begin{pmatrix} -\cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & \cos(2\theta) \end{pmatrix}\Bigg] \end{aligned} \tag{55}$$

where $\Sigma = m_2^2 + m_1^2$ and as usual $\delta m^2 = m_2^2 - m_1^2$. Now an angle χ that rotates $\tilde{H}$ into a diagonal basis such that $H_M = U_M(\chi)\tilde{H}U(\chi)^\dagger_M$ is found to be

$$\tan(2\chi) = \frac{A'\sin(2\psi) + 2\delta m^2\sin(2\theta)}{2\delta m^2\cos(2\theta) - 2A + 2A'\cos(2\psi)} . \tag{56}$$

There are two distinct eigenvalues of H_M

$$\begin{aligned} m_{LK} &= \pm\sqrt{\left(A - \frac{A'}{2}\cos(2\psi) - \delta m^2\cos(2\theta)\right)^2 + \left(\frac{A'}{2}\sin(2\psi) + \delta m^2\sin(2\theta)\right)^2} \\ &+ \Sigma + A - \frac{A'}{2} . \end{aligned} \tag{57}$$

The Eqs. 56 and 57 elucidate why oscillations are possible even for vanishing or degenerate vacuum masses. A non-trivial phase develops and the mass degeneracy is lifted due to the non-standard interaction in the medium, *ie* the effect is proportional to A'.

The validity of performing an analysis of matter enhanced oscillations with NSI using a two neutrino model may be questioned. The approximations that allow the two neutrino analysis to be performed in the standard case need not hold in the presence of non-standard interactions. In the standard oscillation scenario the three neutrino model can be described with two neutrinos for some values of mixings and mass differences. The standard 3×3 mixing matrix is written as the product of three rotations $U = R(\theta_{23}) \cdot R(\theta_{13}) \cdot R(\theta_{12})$. Typically the mass difference δm^2_{21} is taken to vanish approximately. This decouples the $R(\theta_{12})$ rotation, in addition the $R(\theta_{23})$ operates in the 23-subspace and commutes with the matter induced term, meaning that only θ_{13} is enhanced by the induced potential. For the $\nu_e \to \nu_\mu$ channel the standard oscillation probability becomes:

$$P_{e\mu} = \sin^2\theta_{23}\sin^2(2\theta^M_{13})\sin^2(\frac{\delta m^2_{13}}{4E_q}|\vec{L}|) , \tag{58}$$

where θ^M_{13} is the effective mixing angle in matter and δm^2_{13} is also the effective mass difference in matter. Use can be made of Eq. 58 in studying the effects of NSI with the mass difference and effective mixing angle given by Eq. 57 and Eq. 56 if one assumes that the NSI basis is only non-diagonal in the 13 and 12-subsystems of the standard basis. Furthermore one has to assume that the non-standard equivalent of $R(\theta_{12})$ is approximately equal to the identity. For the moment we acknowledge these assumptions as potential pitfalls and perform an analysis for the sake of

gaining an intuitive understanding of the system. The three neutrino scenario will be examined in the next section. For this calculation a standard parameterisation of the $(\nu_\mu N)$ deep inelastic scattering cross section may be used since we are assuming no new physics at the detector, that is [52]

$$\sigma_{\nu_\mu}(E_q) \simeq 0.67 \times 10^{-38} E_q \ \frac{\mathrm{cm}^2}{\mathrm{GeV}} \ . \tag{59}$$

Furthermore the electron neutrino energy distribution has the standard form

$$\frac{d\rho_{\nu_e}}{dx d\Omega_q} = \frac{12}{4\pi} x^2(1-x) \ . \tag{60}$$

The matter induced potential can be written as

$$A = 2\sqrt{2} G_F Y_e \rho E_q = 1.52 \times 10^{-4} \mathrm{eV}^2 Y_e \rho (\mathrm{g/cm}^3) E_q(\mathrm{GeV}) \ , \tag{61}$$

where Y_e is the electron fraction and ρ is the density of matter along the neutrino trajectory. Typically $Y_e \sim 0.5$ in the earth. The value of ρ varies between $\sim 3 - 4.5 \mathrm{g/cm}^3$ depending on the base length and tilt angle of the experiment.

The dependence on ψ is examined in Fig. 3. This plot shows two curves one with $E_\mu = 50$ GeV and the other with $E_\mu = 20$ GeV. The minimum value of NM_{rec} occurs for $\psi = \pi/4$, as one would expect since this angle corresponds to a maximal mixing of non-standard flavours in the standard basis. In fact the behaviour of the diagonalization angle χ for high energy neutrinos also leads one to conclude that the greatest effect will be for $\psi = \pi/4$. For large neutrino energy and $\psi = \pi/4$ Eq. 56 becomes:

$$\begin{aligned} \tan(2\chi) &\approx \frac{A^{'}}{-2A} \\ &= -0.005 \ , \end{aligned} \tag{62}$$

for a non-standard coupling strength of $g^{S_L S_L} = 0.01 g^{LL}$. Without the NSI in the higher energy limit $\tan(2\chi) \to 0$. These results are in contrast to the vacuum oscillations studied in the previous section. The case of new physics at the detector showed the greatest sensitivity when $\psi = (\frac{\pi}{2}, \frac{3\pi}{2})$.

Given that the required detector mass-muon number varies so dramatically with mixing angle one must conclude that it is not wise to make the approximations that allowed the study of this system in terms of two neutrinos only. In addition, note that the effects of NSI during the propagation stage of the oscillation process are about three orders of magnitude greater than for a non-standard interaction of the same strength at either the detection or production stages. This implies that one can study NSI during propagation in isolation of the detector and the source for the matter enhanced channel, an assumption made without justification in previous studies [51, 43, 27].

7 Matter enhanced oscillations—three neutrinos

The cautionary comment of the previous section stating that the assumptions required to treat the NSI in a two neutrino framework is investigated by extending the formalism to allow for three neutrinos. This is done presently, in addition to quantifying the assertion that for matter enhanced oscillations NSI at the source and detector may be neglected.

Extending the formalism of Section 6 presents no real hurdles, but an analytic understanding of the results is now difficult. We find it convenient to put the standard and non-standard interactions with the lepton background on the same footing by evaluating the propagation potential, Eq. 51, in the mass basis. The effective mass of a neutrino in matter is found by evaluating the eigenvalues of the equation of motion in this basis. The unitary matrix, $\tilde{U}_M$, which diagonalizes the effective mass matrix are then found algebraically via a method analogous to that of Ref. [53]. By making the substitution $U^{\alpha\beta} \to \tilde{U}^{\alpha\beta}$, where $\tilde{U}^{\alpha\beta} = U^{\alpha\beta}\tilde{U}_M$, the expression for the event rate at the detector is equivalent to Eq. 37.

We again investigate the case of one additional non-standard interaction, namely a left-chiral scalar interaction. The standard and non-standard mixing matrices, U^{LL} and $U^{S_L S_L}$ respectively,

are parameterised by three different rotation angles, θ_{12}, θ_{13} and θ_{23} for U^{LL} and ϕ_{12}, ϕ_{13} and ϕ_{23} for $U^{S_LS_L}$. The values of the standard mixing angles are taken to be

$$\sin\theta_{12} = 0.53 , \quad (63)$$

$$\sin\theta_{13} = 0.03 , \quad (64)$$

$$\sin\theta_{23} = 0.71 , \quad (65)$$

from Ref. [52]. The numerical evaluation of NM_{rec} proceeds in the same manner as in Sections 5 & 6. When considering muon production at the detector as a result of NSI we make the approximation that $\sigma^{S_LS_L} \sim \sigma^{S_LS_L}$. For the $\nu_e \to \nu_\mu$ channel the interference term in the cross section is ignored due to the comparatively small muon mass.

In Figure 4 & 5 we have plotted the variation of $\mathrm{NM}_{\mathrm{rec}}$ as a function of ϕ_{12} and ϕ_{13} respectively. Note that since we are only considering the effects of the electron background and, for the time being, no new physics at the detector the angle ϕ_{23} decouples from the calculation as per the discussion of the previous section. As was expected the variation of NM_{rec} in Figure 4 takes much the same form as in Figure 3 when $\phi_{12} = 0$, or $R(\phi_{12})$ is diagonal. However when we choose $\phi_{13} = \pi/4$ and vary ϕ_{12} the results are dramatic. While not unexpected these results serve to highlight the point that any experimental analysis aiming to place bounds on the couplings of NSI must consider the full three neutrino system. This applies even for cases where the standard treatment may be done in a two neutrino framework.

Also shown in Figure 4 are the effects of including non-standard physics at the detector in addition to the propagation stage. We see that the assertion made that the results of new physics at the detector are negligible when compared with effects of new physics during propagation are in this case justified.

8 Concluding remarks

In this paper we have united parameterisations of physics beyond the SM such as those of the Michel parameters and non-standard β-decay couplings of precision weak physics experiments with the phenomenon of neutrino oscillations. As such a framework for analysing specific theories, such as Supersymmetry, leptoquarks etc. has been established. This was achieved by developing a scattering theory of the production, propagation and detection processes. In doing so we challenged the notion of what is termed a neutrino flavour state. This was necessitated by the fact that neutrino flavour states do not have a definite mass as is required of intermediate states in a scattering theory, and phenomenologically there is no compelling reason to assume that new interactions couple to the same linear combination of mass states as the V-A interaction. As a result of this formalism we were able to show explicitly the known results that flavour violation or partial flavour violation can not lead to oscillation phenomena in a vacuum. And, in contradistinction, that flavour violation or partial flavour violation in matter can lead to oscillations even for degenerate vacuum masses. The condition for the latter result is that the non-standard interaction be non-diagonal in the standard basis. In addition we have shown that in the ultra-relativistic limit, production and detection processes with opposite chirality decouple, that is right-handed neutrinos become fully sterile from their left-handed counterparts. However in matter a right-handed potential acts as an effective mass linking the two states.

The field theoretic description of neutrino oscillations was used to perform some simple calculations for a generic neutrino factory. This type of experiment was chosen as a convenient starting point due the high intensity and energy of the beam with which the experiments are to be conducted. The beam energy is an important consideration since the field theory was derived in the limit that $\delta m^2|\vec{L}|/2E_q < 1$.

For the vacuum oscillation case we examined the $\nu_\mu \to \nu_\tau$ channel with a left chiral scalar coupling of strength $G^{S_LS_L} = 0.01G^{LL}$. By allowing the non-standard mixing angle, ϕ, to vary we were able to observe flavour conserving and all flavour violating scenarios. We found the greatest effect, at high energies, was for flavour violating interactions which effectively picked out the non-standard term in the cross section. Just above the τ-production threshold flavour conserving interactions play a more important role.

The matter enhanced $\nu_e \to \nu_\mu$ channel was also investigated. Two important differences from the vacuum case were observed. Although the coupling strength of the NSI was the same as that for the one used in the vacuum oscillation study the experimental sensitivity was up to three orders of magnitude greater, indicating that it is safe to ignore new physics at the detector and source for the matter enhanced channels. We also note that the greatest effect when varying the non-standard mixing angle, ψ, did not correspond to direct flavour violation, rather when the non-standard interaction was maximally mixed in the standard basis. In the conventional parlance this would correspond to a decay to a superposition of flavour states.

The focus of this paper has been one of developing a solid framework for future work. Future calculations will concentrate on more realistic experimental simulations than the ones presented here and on performing an analysis of the impact of a right-handed interaction on neutrino propagation through matter.

References

[1] Y. Fukuda et al. *Phys. Rev. Lett.*, **81**:1562–1567, 1998.

[2] M. H. Ahn et al. *Phys. Rev. Lett.*, 90:041801, 2003.

[3] M. S. Neubauer. *Int. J. Mod. Phys.*, **A16S1B**:715–717, 2001.

[4] Q. R. Ahmad et al. *Phys. Rev. Lett.*, **89**:011301, 2002.

[5] Q. R. Ahmad et al. *Phys. Rev. Lett.*, **89**:011302, 2002.

[6] M. C. Gonzalez-Garcia, P. C. de Holanda, Carlos Pena-Garay, and J. W. F. Valle. *Nucl. Phys.*, **B573**:3–26, 2000.

[7] K. Eguchi et al. First results from kamland: Evidence for reactor anti- neutrino disappearance. *Phys. Rev. Lett.*, 90:021802, 2003.

[8] M. Apollonio et al. *Phys. Lett.*, **B466**:415–430, 1999.

[9] Y. Itow et al. The jhf-kamioka neutrino project. *hep-ex/***0106019**, 2001.

[10] D. Finley and N. Holtkamp. *Nucl. Instrum. Meth.*, **A472**:388–394, 2000.

[11] Charles M. Ankenbrandt et al. *Phys. Rev. ST Accel. Beams*, **2**:081001, 1999.

[12] M. Fritschi et al. *Phys. Lett.*, **B173**:485–489, 1986.

[13] R. G. H. Robertson et al. *Phys. Rev. Lett.*, **67**:957–960, 1991.

[14] A. I. Belesev et al. *Phys. Lett.*, **B350**:263–272, 1995.

[15] V. M. Lobashev et al. *Phys. Lett.*, **B460**:227–235, 1999.

[16] C. Weinheimer et al. *Phys. Lett.*, **B300**:210–216, 1993.

[17] Andre Rouge. *Eur. Phys. J.*, **C18**:491–496, 2001.

[18] W. Fetscher and H. J. Gerber. *Eur. Phys. J.*, **C15**:316–318, 2000.

[19] J. D. Jackson et al. *Phys. Rev.*, **106**:517, 1957.

[20] C. Weinheimer et al. *Phys. Lett.*, **B460**:219–226, 1999.

[21] B. H. J. McKellar, M. Garbutt, G. J. Stephenson, and T. Goldman. Neutrino masses or new interactions. *hep-ph/***0106122**, 2001.

[22] G. J. Stephenson, T. Goldman, and B. H. J. McKellar. *Phys. Rev.*, **D62**:093013, 2000.

[23] G. J. Stephenson and T. Goldman. *Phys. Lett.*, **B440**:89–93, 1998.

[24] Y. Farzan, O. L. G. Peres, and A. Yu. Smirnov. *Nucl. Phys.*, **B612**:59–97, 2001.

[25] Toshihiko Ota, Joe Sato, and Nao-aki Yamashita. *Phys. Rev.*, **D65**:093015, 2002.

[26] Loretta M. Johnson and Douglas W. McKay. *Phys. Rev.*, **D61**:113007, 2000.

[27] P. Huber, T. Schwetz, and J. W. F. Valle. Confusing non-standard neutrino interactions with oscillations at a neutrino factory. *hep-ph/***0202048**.

[28] L. Wolfenstein. *Phys. Rev.*, **D17**:2369, (1978).

[29] S. P. Mikheev and A. Yu. Smirnov. *Nuovo Cim.*, **C9**:17, (1986).

[30] R. N. Mohapatra and P. B. Pal. Massive neutrinos in physics and astrophysics. second edition. *World Sci. Lect. Notes Phys.*, **60**:1–397, 1998.

[31] Christian Y. Cardall and Daniel J. H. Chung. *Phys. Rev.*, **D60**:073012, 1999.

[32] Mark J. Thomson and Bruce H. J. McKellar. *Phys. Lett.*, **B259**:113–118, 1991.

[33] Bruce H. J. McKellar and Mark J. Thomson. *Phys. Rev.*, D49:2710–2728, 1994.

[34] R. N. Mohapatra. *Unification and Supersymmetry. The frontiers of quark-lepton physics.* Berlin, Germany: Springer (1986) 309 P. (Contemporary Physics).

[35] Anindya Datta, Raj Gandhi, Biswarup Mukhopadhyaya, and Poonam Mehta. *Phys. Rev.*, **D64**:015011, 2001.

[36] Jogesh C. Pati and Abdus Salam. *Phys. Rev.*, **D10**:275–289, 1974.

[37] Mikael Beuthe. *Phys. Rept.*, 375:105–218, 2003.

[38] W. Grimus, P. Stockinger, and S. Mohanty. *Phys. Rev.*, **D59**:013011, 1999.

[39] W. Grimus and P. Stockinger. *Phys. Rev.*, **D54**:3414–3419, 1996.

[40] Ken Kiers and Nathan Weiss. *Phys. Rev.*, **D57**:3091–3105, 1998.

[41] C. Giunti, C. W. Kim, J. A. Lee, and U. W. Lee. *Phys. Rev.*, **D48**:4310–4317, 1993.

[42] Sven Bergmann, Yuval Grossman, and Enrico Nardi. *Phys. Rev.*, **D60**:093008, (1999).

[43] P. Huber and J. W. F. Valle. *Phys. Lett.*, **B523**:151, (2001).

[44] P. Herczeg. *Prog. Part. Nucl. Phys.*, **46**:413–457, 2001.

[45] P. Herczeg. Beta decay and muon decay beyond the standard model. 1996. In Langacker, P. (ed.): Precision tests of the standard electroweak model* 786-837.

[46] B. Kayser, F. Gibrat-Debu, and F. Perrier. The physics of massive neutrinos. *World Sci. Lect. Notes Phys.*, **25**:1–117, 1989.

[47] M. A. G. Aivazis, Frederick I. Olness, and Wu-Ki Tung. *Phys. Rev.*, **D50**:3085, (1994).

[48] C. H. Albright and C. Jarlskog. *Nucl. Phys.*, **B84**:467, (1975).

[49] Lawrence J. Hall and Hitoshi Murayama. *Phys. Lett.*, **B463**:241, (1999).

[50] E. Leader and E. Predazzi. An introduction to gauge theories and modern particle physics. vol. 2. *Cambridge Monogr. Part. Phys. Nucl. Phys. Cosmol.*, 4:1–431, 1996.

[51] N. Fornengo, M. Maltoni, R. Tomas Bayo, and J. W. F. Valle. *Phys. Rev.*, **D65**:013010, (2002).

[52] M. Freund, P. Huber, and M. Lindner. *Nucl. Phys.*, **B585**:105, (2000).

[53] G. Cheng. Mixing of three neutrinos in matter. *hep-ph/***0104042**, 2001.

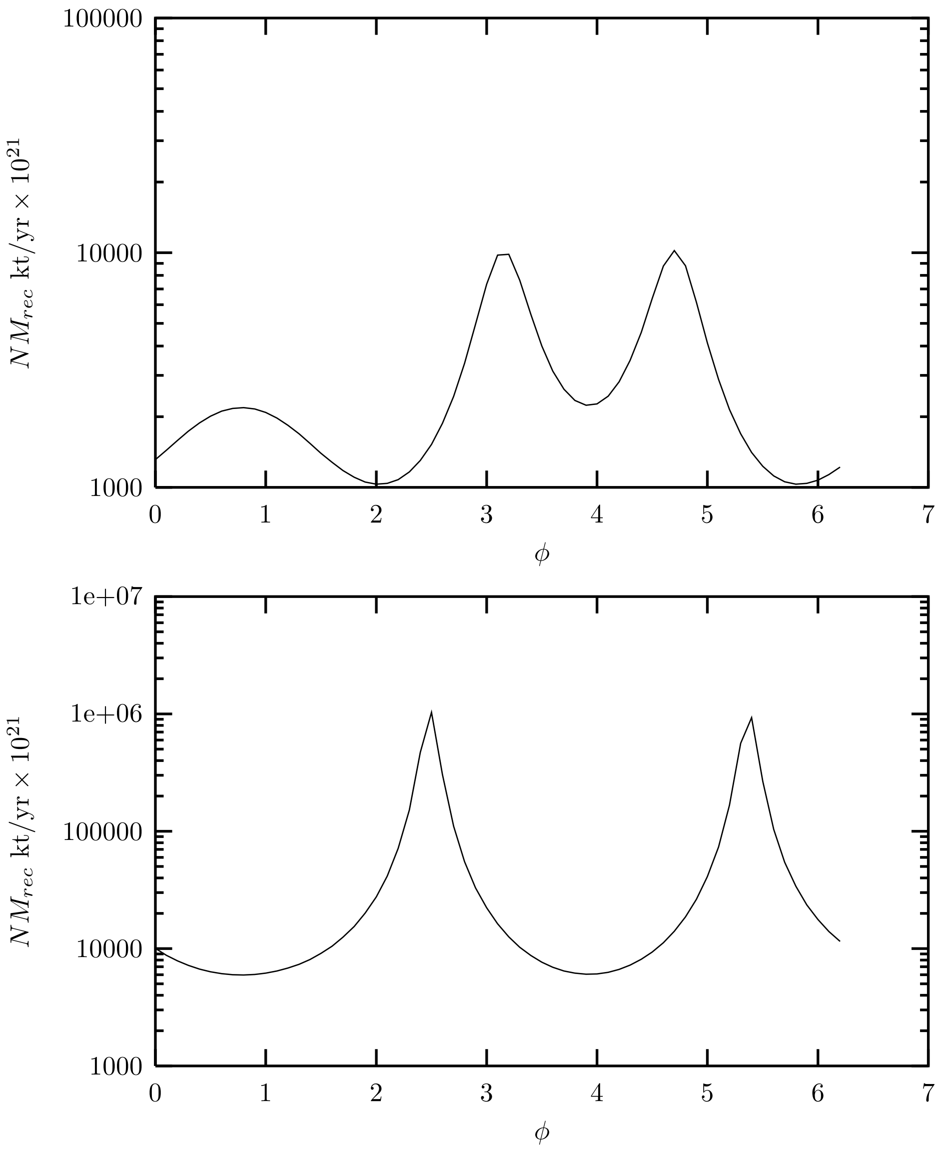
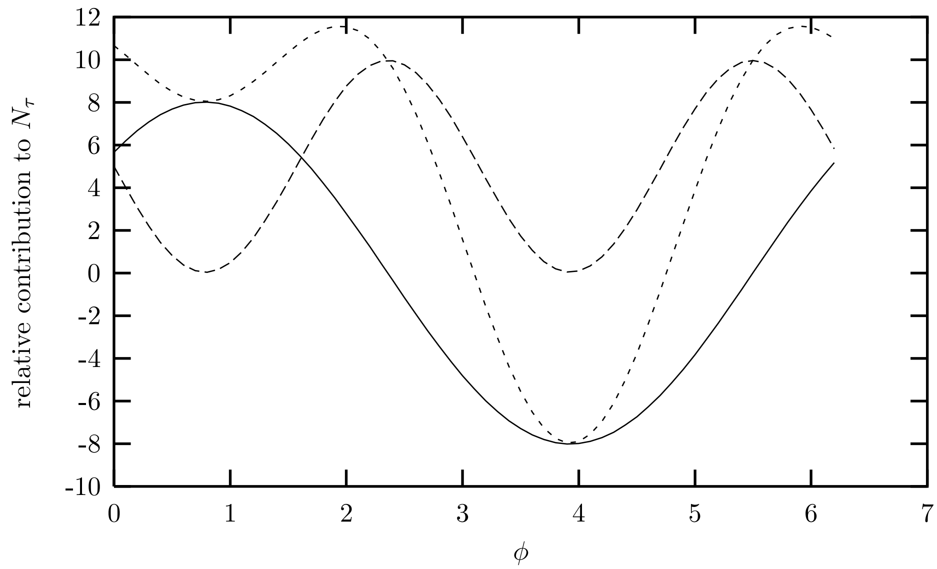


Figure 1: Top panel: The required number of detector mass-muon number units NM_{rec} with $G^{S_L S_L} = 0.01 G^{LL}$, $\delta m^2 = 0.0025 eV^2$, $L = 732 km$. Bottom panel: The required number of detector mass-muon number units NM_{rec} with $G^{S_L S_L} = 0.01 G^{LL}$, $\delta m^2 = 0.0025 eV^2$, $L = 732 km$.



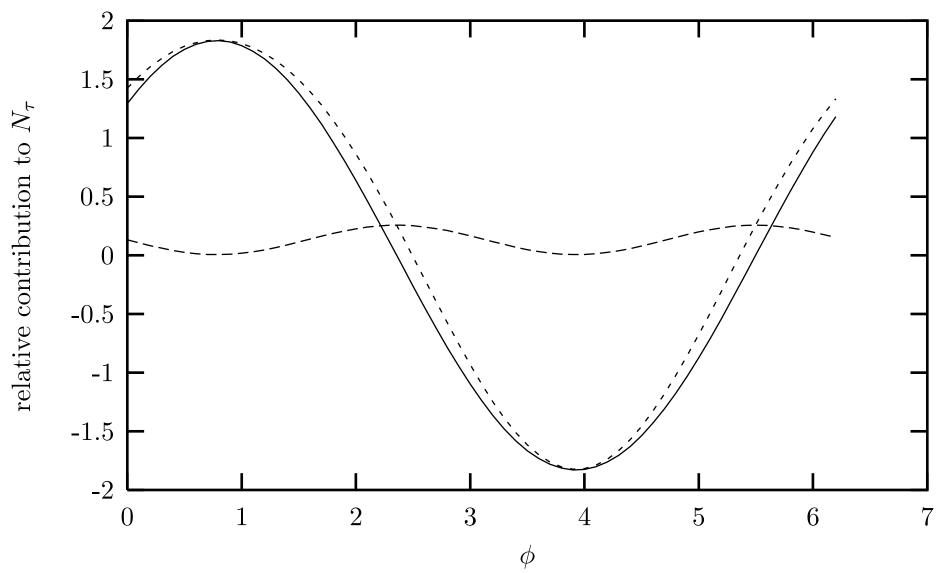


Figure 2: Top panel: Relative contribution to the charged current event rate for the pure scalar term, long dash, the interference term, solid, and the sum of the two, short dash. With $G^{S_L S_L} = 0.01 G^{LL}$, $\delta m^2 = 0.0025 eV^2/c^4$, $L = 732 km$ and $E_\mu = 50$ GeV. Bottom panel: Relative contribution to the charged current event rate for the pure scalar term, long dash, the interference term, solid, and the sum of the two, short dash. With $G^{S_L S_L} = 0.01 G^{LL}$, $\delta m^2 = 0.0025 eV^2/c^4$, $L = 732 km$ and $E_\mu = 20$ GeV.

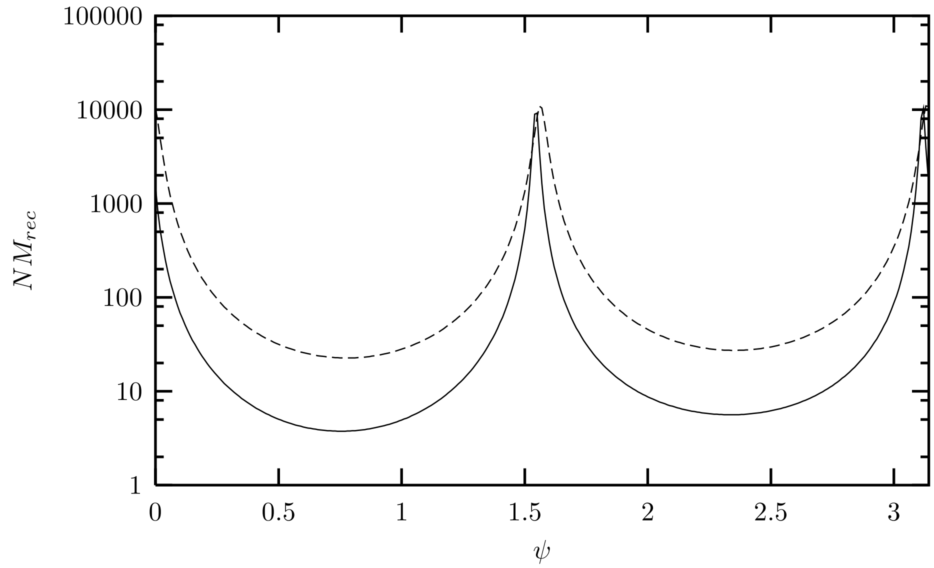


Figure 3: The required detector mass-muon number in units of 2×10^{21} kt/yr. The solid line is with $E_\mu = 50$ GeV, the long-dashed line is with $E_\mu = 20$ GeV. The vacuum mass difference is taken to be positive with $|\delta m^2| = 3.5 \times 10^{-3}\mathrm{eV}^2/\mathrm{c}^4$, $|\vec{L}| = 7332$km and $\sin^2(2\theta) = 0.004$.

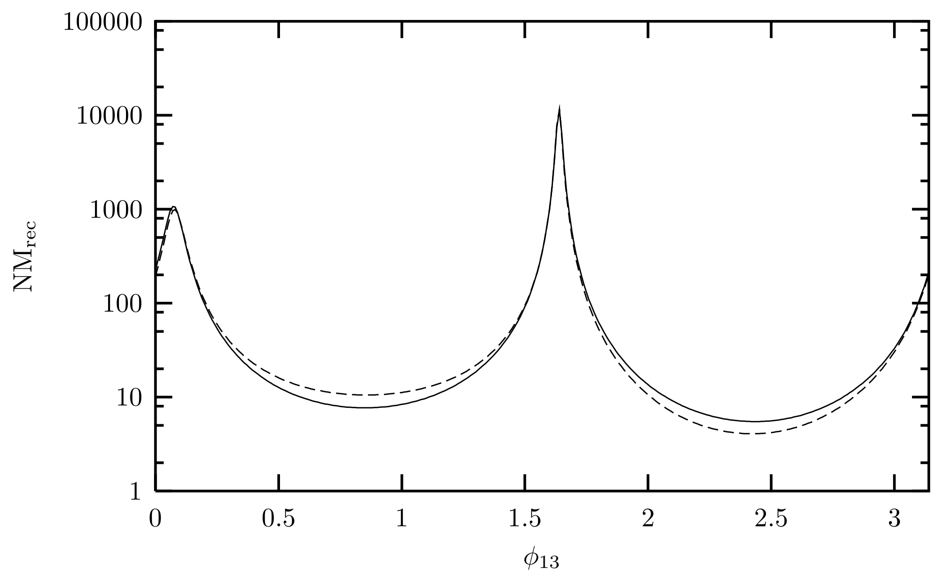


Figure 4: The required detector mass-muon number in units of 2×10^{21} kt/yr and for$E_\mu = 50$ GeV, $L = 7332$ km, $\phi_{12} = 0$, $\phi_{23} = \theta_{23}$ and $g^{S_L S_L} = 0.01 g^{LL}$. The solid line is the event rate with no new physics at the detector. While the dashed line is the event rate with a non-standard coupling of $G^{S_L S_L} = 0.01 G^{LL}$ at the detector.

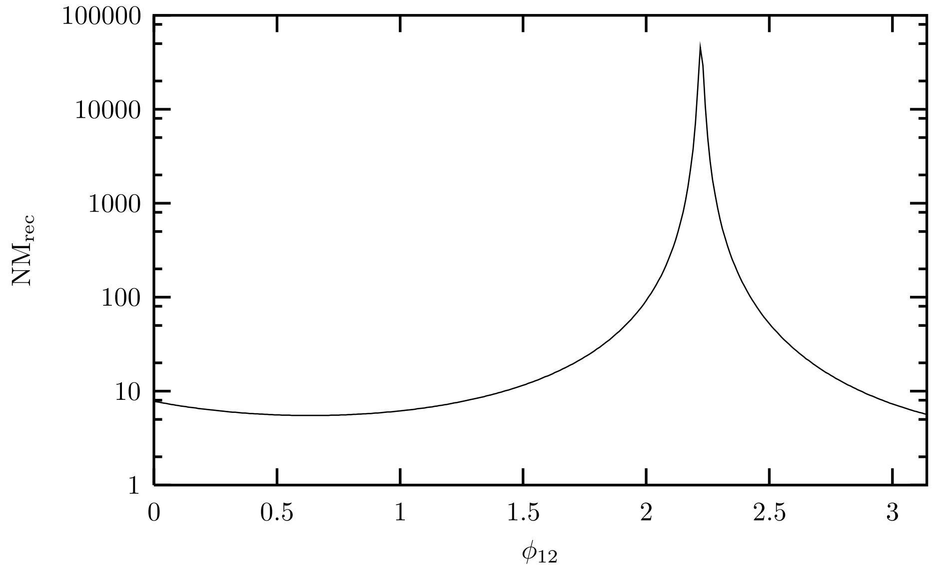


Figure 5: The required detector mass-muon number in units of 2×10^{21} kt/yr as a function of mixing angle ϕ_{12}. The other parameters are $E_\mu = 50$ GeV, $L = 7332$ km, $\phi_{13} = \pi/4$, $\phi_{23} = \theta_{23}$ and $g^{S_L S_L} = 0.01 g^{LL}$. In this plot we have assumed no new physics at the detector.